Information Propagation in Stochastic Networks

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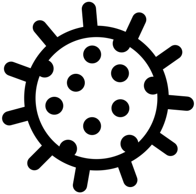
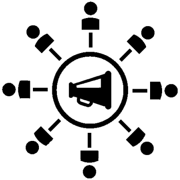
Some Applications of Information Propagation



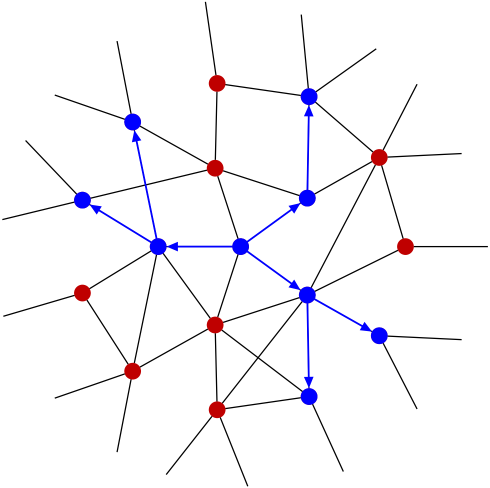
Epidemiology

Bitcoin



Marketing

Propagation of a Piece of Information



A Baseline Model

$$\partial_t \rho_k = \mu_{\mathrm{SI}} k \left(1 - \rho_k\right) \Theta$$

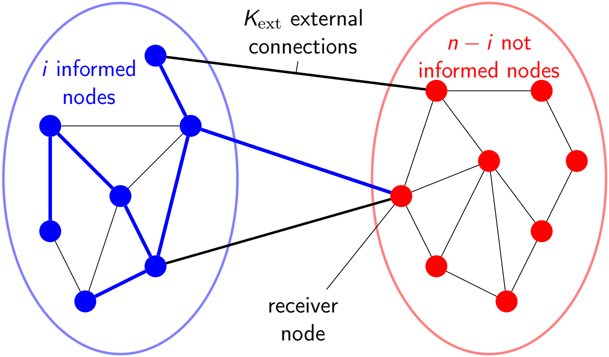
$$\Theta = \frac{\sum_{k'} \rho_{k'} k' P_{k_{\mathrm{tot}}}(k')}{\sum_{k'} k' P_{k_{\mathrm{tot}}}(k')}$$

- Spanning tree is not considered
- Inaccurate in sparse networks in the initial phase

Goal

- Goal
  - Create a more accurate model for sparse networks
  - Verify the model
- Challenge
  - Application of complex calculations
  - No real-world measurement data
- Solution
  - Take into account the spanning tree
  - Develop a probabilistic model
  - Implement a Monte-Carlo network simulation

Notations


K_{ext} external connections
i informed nodes
$n-i$ not informed nodes
receiver node

Differential Equation for the Informed Nodes

$$\frac{\Delta t}{\Delta i} \sim \mathrm{Exponential}\left(\mu K_{\mathrm{ext}}\right)$$

Change of the

$$\left.\begin{aligned} \mathbb{E}\left[\frac{\Delta t}{\Delta i}\right] &= \frac{1}{\mu}\mathbb{E}\left[\frac{1}{K_{\mathrm{ext}}}\right] \\ \mathbb{E}\left[\frac{1}{K_{\mathrm{ext}}}\right] &\approx \frac{1}{\mathbb{E}K_{\mathrm{ext}}}\left(1+\frac{\mathbb{D}^2 K_{\mathrm{ext}}}{\mathbb{E}^2 K_{\mathrm{ext}}}+\cdots\right) \end{aligned}\right\} \longrightarrow \frac{\mathrm{d}\,\mathbb{E}t}{\mathrm{d}i} = \frac{1}{\mu}\frac{1}{\mathbb{E}K_{\mathrm{ext}}}$$

Differential Equation for the External Connections

$$\frac{\Delta K_{\text{ext}}}{\Delta i} = k_{\text{recv}} - 2k_{\text{recv}}^{\text{inf}} \quad \longrightarrow \quad \frac{\mathbb{E}\left[\Delta K_{\text{ext}}\right]}{\Delta i} = \mathbb{E}k_{\text{recv}} - 2\mathbb{E}k_{\text{recv}}^{\text{inf}}$$

$$\mathbb{E}k_{\text{recv}}^{\text{inf}} = 1 + \left(k_{\text{recv}} - 1\right) \frac{K_{\text{ext}}}{K_{\text{ext}} + K_{\text{tot ni}}}$$

Degree Distribution of the Informed Nodes

Degree distribution of the receiver node:

$$P_{k_{\mathrm{recv}}}(k \mid i) = \frac{k}{\mathbb{E}k_{\mathrm{ninf}}} P_{k_{\mathrm{ninf}}}(k \mid i)$$

Degree distribution of the not informed nodes:

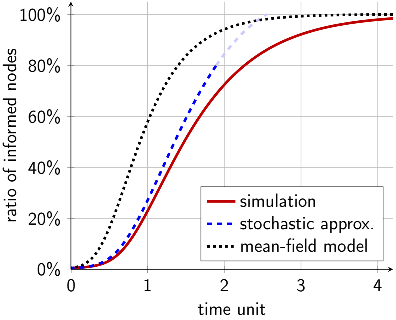
$$\frac{\partial P_{k_{\mathrm{ninf}}}(k \mid i)}{\partial i} = \frac{P_{k_{\mathrm{ninf}}}(k \mid i)}{n - i} \left(1 - \frac{k}{\mathbb{E}k_{\mathrm{ninf}}}\right)$$

Pseudocode of Monte-Carlo Simulation

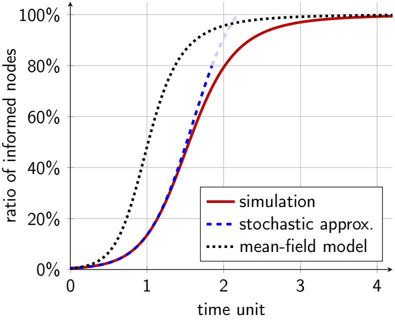
```
while remaining simulations do
    build network
        create nodes;
        assign degrees to nodes;
        connect nodes randomly
            create array of free degrees;
            while new connections possible do
                create random connection;
            end
    initialize propagation
        create event queue;
        inject information to a random node;
        create new events for each neighbor;
    simulate propagation
        while event queue is not empty do
            get next event;
            adjust time based on next event;
            if target node is not informed then
                inform target node;
                get list of neighbors;
                update event queue with new events;
            end
        end
    save results
end
```

https://github.com/shepherd92/inf_prop_simulator

Comparison With the Mean-Field Model



Scale-free network



Erdős–Rényi network

Summary

- Goal: create and verify a more accurate model for sparse networks compared to the SI mean-field model
- Solution: take into account the spanning tree connecting the informed nodes
- Results are verified through Monte-Carlo network simulation

Thank you for your attention!

reference: Juhász, P. L. (2021). Information Propagation in Stochastic Networks. *Physica A: Statistical Mechanics and its Applications 577*, 126070.